

Conductance of Tomonaga-Luttinger liquid wires and junctions with resistances

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Abstract. - We study the effect that resistive regions have on the conductance of a quantum wire with interacting electrons which is connected to Fermi liquid leads. Using the bosonization formalism and a Rayleigh dissipation function to model the power dissipation, we use both scattering theory and Green's function techniques to derive the DC conductance. The resistive regions are generally found to lead to incoherent transport. For a single wire, we find that the resistance adds in series to the contact resistance of h/e^2 for spinless electrons, and the total resistance is independent of the Luttinger parameter K_W of the wire. We numerically solve the bosonic equations to illustrate what happens when a charge density pulse is incident on the wire; the results depend on the parameters of the resistive and interacting regions in interesting ways. For a junction of Tomonaga-Luttinger liquid wires, we use a dissipationless current splitting matrix to model the junction. For a junction of three wires connected to Fermi liquid leads, there are two families of such matrices; we find that the conductance matrix generally depends on K_W for one family but is independent of K_W for the other family, regardless of the resistances present in the system.

Introduction. - It is well known that for non-interacting electrons, the conductance of a narrow ballistic quantum wire is quantized in units of e^2/h at low temperatures [1,2]. This remains true when electron-electron interactions are taken into account in the wire, provided that there are no sources of backscattering (such as impurities) and that the wire is connected to leads where there are no interactions [3–7]. Namely, if the wire is modeled as a Tomonaga-Luttinger liquid (TLL) and the interaction strength is given by the Luttinger parameter K_W , the conductance of a clean wire is independent of K_W . This breaks down if there are isolated impurities in a wire with interacting electrons; the impurity strengths then satisfy some renormalization group (RG) equations, and the conductance depends on K_W and other parameters like the wire length, the distances between the impurities, and the temperature [8–10]. One can think of the impurities as giving rise to a resistance which leads to power dissipation, although this aspect is usually not highlighted in the literature. There have been some studies of power dissipation on the edges of a quantum Hall system [11] and also at a junction of quantum wires due to the presence of bound states [12]. However, there has been relatively little discus-

sion of the effects of an extended region of dissipation (a patch of resistance) within the framework of TLL theory or bosonization which is well-suited for studying the effects of interactions between electrons [13]. Such a theory would have the benefit of combining the wealth of knowledge of TLLs with the classical notion of resistance. Further, a large amount of work has been done on junctions of several quantum wires theoretically [14–23] and experimentally [24,25], and it would be useful to know what effect resistances in the wires have on the conductance matrix of such a system. A junction of three quantum wires with interacting spin-1/2 electrons has been studied in Ref. [26], and it has been found that some of the fixed points of the RG equations have different properties for the charge and spin sectors. In this context, we would like to mention the work in Ref. [27]. Here the effect of an extended region of inhomogeneity in a quantum wire has been studied, and it has been shown that this leads to weak backscattering which gives rise to a resistance which is linear in the temperature. Further, the resistances for the charge and spin sectors are different; the sum of the two gives the total resistance.

In this paper, we will use the technique of bosoniza-

tion to study the effect of patches of resistance on the conductance of a quantum wire system with or without junctions. Our treatment will be classical in the sense that the resistance will be taken to be purely a source of Ohmic power dissipation; we will not consider the microscopic origins of the resistance such as point impurities which can scatter the electrons quantum mechanically. As a result, the transport will be seen to be incoherent, with the resistance of different patches adding in series with no effects of interference; the incoherence also implies that RG equations will play no role in the analysis. Using the idea of a Rayleigh dissipation function [28] to model the resistance patches, we will obtain the equations of motion for the bosonic field whose space and time derivatives give the electron charge density and current respectively. We then use both a scattering solution of the equations of motion [5, 6] and a Green's function approach [3] to obtain the DC conductance G . Our analysis leads to several new results which are as follows. For a single wire, we calculate G when the Luttinger parameter K , the velocity v and the resistivity r all vary with the spatial coordinate x in the region of the quantum wire. The equations of motion enable us to numerically study the space-time evolution when a charge density wave of arbitrary shape is incident on the wire region. For the case of several quantum wires meeting at a junction, we model the junction using an orthogonal and dissipationless current splitting matrix M . For a three-wire junction, it is known that there are two families of M which have determinant ± 1 respectively. When resistance patches are then introduced in each of the wires some distance away from the junction, the conductance matrix G of the system is generally found to depend on the matrix M as well as some of the parameters mentioned above; this will be discussed in detail. For simplicity, we will restrict our analysis to the case of spinless electrons.

Equation of motion.- We begin by studying a single wire with interacting spinless electrons. In the absence of backscattering processes, the bosonic Lagrangian is given by

$$L = \int_{-\infty}^{\infty} dx \left[\frac{1}{2vK} (\partial_t \phi)^2 - \frac{v}{2K} (\partial_x \phi)^2 \right], \quad (1)$$

where K and v denote the Luttinger parameter and velocity respectively; these parameters can vary with x within a finite region which we will take to be $-L/2 < x < L/2$. The Fermi liquid leads will be assumed to lie in the regions $|x| > L/2$, where $v = v_F$ and $K = 1$ are constant; in the leads, the frequency and wave number of a plane wave are related as $\omega = v_F |k|$. The electron charge density n and current j are given in terms of the bosonic field as $n = -e \partial_x \phi / \sqrt{\pi}$ and $j = e \partial_t \phi / \sqrt{\pi}$, where e is the electron charge; these densities clearly satisfy the equation of continuity $\partial_t n + \partial_x j = 0$. The energy of the system is then given by

$$E = \int_{-\infty}^{\infty} dx \left[\frac{1}{2vK} (\partial_t \phi)^2 + \frac{v}{2K} (\partial_x \phi)^2 \right]. \quad (2)$$

We now introduce dissipation in the model through a Rayleigh dissipation function

$$\mathcal{F} = \frac{1}{2} \int_{-\infty}^{\infty} dx \, r \, j^2, \quad (3)$$

where the resistivity r can also vary with x but will be taken to be non-zero only within the region $|x| < L/2$. The function in eq. (3) contributes to the equation of motion as $d/dt(\delta L / \delta \partial_t \phi) - \delta L / \delta \phi + \delta \mathcal{F} / \delta \partial_t \phi = 0$ [28], which gives

$$\frac{1}{vK} \partial_t^2 \phi - \partial_x \left(\frac{v}{K} \partial_x \phi \right) + \frac{e^2}{\pi} r \partial_t \phi = 0. \quad (4)$$

(Note that we have set $\hbar = 1$, so that $e^2/(2\pi) = e^2/h$). One can then show that the power dissipation is given by

$$\frac{dE}{dt} = - \int_{-\infty}^{\infty} dx \, \partial_t \phi \, \delta \mathcal{F} / \delta \partial_t \phi \quad (5)$$

which is equal to $-j^2 R$ in a steady state as desired; here $R = \int_{-\infty}^{\infty} dx \, r$ is the total resistance, and steady state means that j is independent of x (this follows from the equation of continuity and the fact that $\partial_t n = 0$ in a steady state).

One can compute the conductance of the system in two ways. The first way is to consider the interacting and resistive regions as sources of scattering, as has been done for an interacting region in Refs. [5, 6]. We allow a plane wave with frequency ω to be incident on this region from the left, and compute the reflection and transmission amplitudes as functions of ω . The latter amplitude is related, in the limit $\omega \rightarrow 0$, to the dc conductance σ_{dc} . The second way is to compute the Fourier transform of the Green's function in imaginary time and hence the nonlocal ac conductance; this again gives σ_{dc} in the limit $\omega \rightarrow 0$ [3]. We will use both these methods, the scattering method for a single wire and the Green's function for a junction of three wires.

Transmission through a dissipative region.- To illustrate the scattering method for computing σ_{dc} [5, 6], let us first consider a non-interacting system in which $K = 1$ and $v = v_F$ are independent of x , while $r(x) = r_0$ for $-a < x < a$ and 0 elsewhere. This describes a dissipative region $(-a, a)$ connected to leads on the two sides. For a plane wave incident from the left with $k = \omega/v_F$, the spatial part of the solution $\phi_k(x, t) = f_k(x) e^{-i\omega t}$ is given by

$$f_k = e^{ikx} + s_k e^{-ikx} \quad \text{for } x \leq -a, \quad (6a)$$

$$= t'_{k'} e^{ik'x} + s'_{k'} e^{-ik'x} \quad \text{for } -a \leq x \leq a, \quad (6b)$$

$$= t_k e^{ikx} \quad \text{for } x \geq a. \quad (6c)$$

Using eq. (4), and the continuity of f_k and $\partial_x f_k$ at $x = \pm a$,

we find that s_k and t_k are given by

$$\begin{aligned} s_k &= \frac{(\eta^2 - 1)(e^{i2\eta ka} - e^{-i2\eta ka})}{(1 + \eta)^2 - (1 - \eta)^2 e^{i4\eta ka}}, \\ t_k &= \frac{4\eta e^{i2ka(\eta-1)}}{(1 + \eta)^2 - (1 - \eta)^2 e^{i4\eta ka}}, \end{aligned} \quad (7)$$

$$\text{where } \eta = \frac{k'}{k} = \sqrt{1 + \frac{\zeta}{k}}, \text{ and } \zeta = i\frac{e^2}{\pi}r_0. \quad (8)$$

We now consider what happens when a δ -function charge density pulse is incident on the dissipative region from the left lead. At $t = 0$, the pulse is at $x_0 < -a$ with velocity $+v_F$. This pulse is described by:

$$\phi(x, t) = i\sqrt{\pi} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{e^{ik(x-x_0-v_F t)}}{k + i\epsilon}, \quad (9)$$

so that $n = e\delta(x - x_0 - v_F t)$ and $j = ev_F \delta(x - x_0 - v_F t)$ for $x < -a$ and $t < -(a + x_0)/v_F$ (before scattering into the resistive region). For $x > a$, the corresponding current is given by

$$j(x, t) = ev_F \int_{-\infty}^{\infty} \frac{dk}{2\pi} t_k e^{ik(x-x_0-v_F t)}. \quad (10)$$

The nonlocal ac conductivity is given by $\sigma(x, x_0, t) = ej(x, t)/(2\pi)$, where $x > a$ and $x_0 < -a$. The DC conductance is given by the zero frequency limit of the Fourier transform,

$$\begin{aligned} \sigma_{dc} &= \lim_{\Omega \rightarrow 0^+} \int_{-\infty}^{\infty} dt e^{i\Omega t} \sigma(x, x_0, t) \\ &= \frac{e^2}{2\pi} \lim_{k \rightarrow 0^+} t_k \\ &= \frac{e^2}{2\pi} \lim_{k \rightarrow 0^+} \left[\frac{4\eta}{(1 + \eta)^2} \frac{e^{i2(\eta-1)ka}}{1 - \chi e^{i4\eta ka}} \right], \end{aligned} \quad (11)$$

where $\chi = (1 - \eta)^2/(1 + \eta)^2$. Using $\lim_{k \rightarrow 0} \eta = \sqrt{\zeta/k}$ and $\lim_{k \rightarrow 0} \chi = 1 - 4\sqrt{k/\zeta}$, we find that

$$\sigma_{dc} = \frac{e^2}{2\pi} \frac{1}{1 - i\zeta a} = \frac{e^2}{2\pi} \frac{1}{1 + \frac{e^2 R}{2\pi}}, \quad (12)$$

where $R = 2ar_0$ is the total resistance. This expression shows that R adds in series to the contact resistance of $2\pi/e^2$; this is the expected property of resistance in a phase incoherent system. We obtain the same result for σ_{dc} through the Green's function method outlined below for a three-wire system.

The expression in eq. (12) can be derived for a general resistance profile $r(x)$ as follows. For $\omega = 0$, a solution of eq. (4) is $\phi = c$, where c is a constant. Let us now look for a solution which is valid upto first order in ω and $k = \omega/v_F$, and which reduces to $\phi = c$ in the limit $\omega \rightarrow 0$. Assuming that $\phi_k(x, t) = f_k(x)e^{-i\omega t}$, where f_k has the forms given by eq. (6a) and eq. (6c) for $x < -a$ and $> a$ respectively, we must have $1 + s_k = t_k = c$ to zero-th order

in ω as $\omega \rightarrow 0$. Next, on ignoring the term of order ω^2 in eq. (4), we obtain

$$- \partial_x \left(\frac{v}{K} \partial_x f_k \right) - i\omega \frac{e^2}{\pi} r f_k = 0. \quad (13)$$

Replacing f_k by the constant $c = t_k$ in the second term in eq. (13) (we can do this since that term has a factor of ω), and integrating that equation from $x = -a - \epsilon$ to $a + \epsilon$, we obtain $ikv_F[t_k - (1 - s_k)] = -i\omega t_k(e^2 R/\pi)$, where $R = \int_{-a}^a dx r(x)$ is the total resistance. Combining this with $1 + s_k = t_k$, we obtain

$$t_{k \rightarrow 0} = \frac{1}{1 + \frac{e^2 R}{2\pi}}, \quad (14)$$

from which the result for $\sigma_{dc} = (e^2/2\pi)t_{k \rightarrow 0}$ follows.

It is interesting to compare the evolution of a charge density pulse incident on a dissipative region in a non-interacting system with the evolution of the same pulse in a non-dissipative but interacting system ($K_W \neq 1$). [The latter case was studied in Refs. [5,6]. It was shown there that a series of pulses emerges on both sides of the wire, such that eventually the integrated pulse on the left (i.e., the total reflection probability) is zero, while the integrated pulse on the right (i.e., the total transmission probability) is unity]. We have time evolved eq. (4) numerically; a von Neumann stability analysis was performed to ensure that the numerical errors remain small.

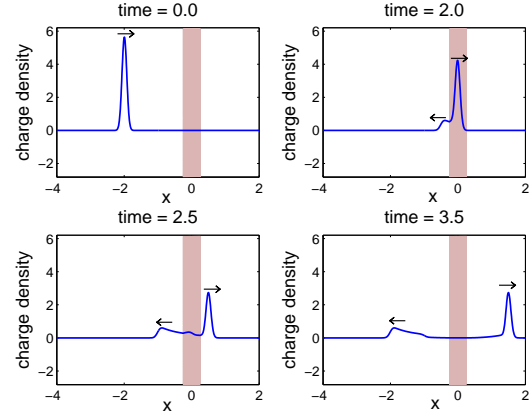


Fig. 1: Evolution of a charge density pulse incident on a dissipative region (shaded), with $l_0 = 0.1$ (the width of the pulse), $a = 0.25$ and $r_0 = 3$.

Figures 1 and 2 show the density profiles at different times for the purely dissipative and purely interacting cases respectively; in the first case, we have chosen $K = 1$ and $v = 1$ everywhere, while in the second case, we have chosen $K = 1$ and $v = 1$ in the leads, but $K = 0.6$ and $v = 1.6$ in the interacting region. In fig. 1, one sees only one reflected and one transmitted pulse. The width of the reflected pulse ($= 4a$) is equal to twice the length of the dissipative region, providing us with an insight into the

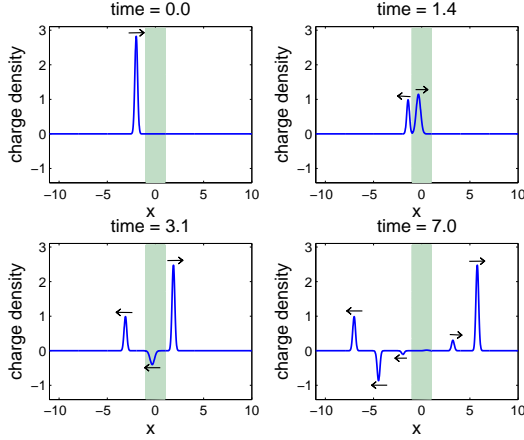


Fig. 2: Evolution of a charge density pulse incident on an interacting region (shaded), with $l_0 = 0.2$ (the width of the pulse), $a = 1$, $K_W = 0.6$ and $v_W = 1.6$.

nature of dissipation. Namely, a pulse gets reflected from each point in a dissipative region; this makes the width of the reflected pulse (i.e., the distance between waves being reflected from the left and right ends of that region) equal to $4a$. On the other hand, when a pulse approaches an interacting region in which K is piecewise constant, it gets reflected only from the points of inhomogeneity, i.e., where dK/dx is not zero. Fig. 2 shows a series of reflected and transmitted pulses in agreement with the results obtained analytically in Refs. [5, 6].

Green's function calculation for three-wire junction.— We now consider a junction of three dissipative TLL wires as shown in fig. 3 (a), each of which contains three regions: (i) $0 \leq x_i \leq L_{i1} \neq 0$ — the region around the junction where $K(x_i) = K_W$; elsewhere $K(x_i) = 1$, (ii) $L_{i1} \leq x_i \leq L_{i2}$ — a dissipative region where $r(x_i) = r_{i0}$; elsewhere $r(x_i) = 0$, and (iii) $x_i \geq L_{i2}$ — semi-infinite leads.

Here i labels the wires, and on wire i , the coordinate x_i runs from 0 to ∞ , with $x_i = 0$ corresponding to the junction point. The regions $x_i \geq L_{i2}$ model the two- or three-dimensional leads which are assumed to be Fermi liquids with no interactions between the electrons; hence we set $K = 1$ in those regions.

Following Ref. [3], we write

$$I_i = \sum_{j=1}^3 \int_0^{L_{j2}} dx'_j \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sigma_{ij,\omega}(x_i, x'_j) E_\omega(x'_j), \quad (15)$$

in the linear response regime, where $E_\omega(x'_j)$ is the Fourier component of the electric field $E(x'_j, t)$ on wire j , and $\sigma_{ij,\omega}(x_i, x'_j)$ is the nonlocal conductance matrix. We then obtain

$$\sigma_{ij,\omega}(x_i, x'_j) = -\frac{e^2 \bar{\omega}}{\pi} \mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j), \quad (16)$$

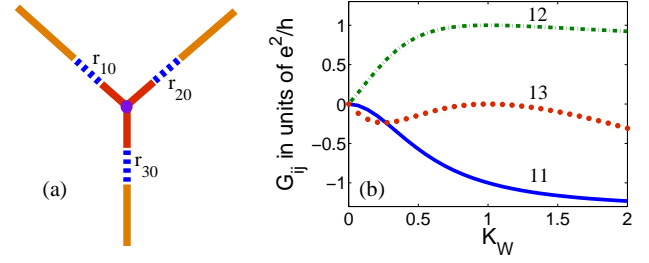


Fig. 3: (a) A schematic diagram of a three-wire junction with an interacting region close to the junction (red solid region), a dissipative region (blue dotted region further from the junction), and Fermi liquid leads (brown solid region furthest from the junction). (b) The matrix elements G_{ij} , with $ij = 11, 12$ and 13 , are shown as functions of K_W for the case of M_1 with $\theta = 2\pi/3$ and $r_{i0} = 0$ for $i = 1, 2, 3$.

where $\bar{\omega} = -i\omega$, and

$$\mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j) = \int_{-\infty}^{\infty} \frac{d\tau}{2\pi} \langle T_\tau^* \phi_i(x_i, \tau) \phi_j(x'_j, 0) \rangle e^{-i\bar{\omega}\tau} \quad (17)$$

is the propagator of the bosonic field in imaginary time, $\tau = it$. The Green's function satisfies the equation

$$\left[-\partial_{x_i} \left(\frac{v(x_i)}{K(x_i)} \partial_{x_i} \right) + \frac{\bar{\omega}^2}{v(x_i)K(x_i)} - \frac{e^2 \bar{\omega}}{\pi} r(x_i) \right] \mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j) = \delta_{ij} \delta(x_i - x'_j), \quad (18)$$

with the following boundary conditions:

- (i) $\mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j)$ is continuous at $x_i = x'_j$ (where $0 < x'_j < L_{j2}$) and $-\frac{v(x_i)}{K(x_i)} \partial_{x_i} \mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j) \Big|_{x'_j - \epsilon}^{x'_j + \epsilon} = \delta_{ij}$,
- (ii) $\mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j)$ and $-\frac{v(x_i)}{K(x_i)} \partial_{x_i} \mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j)$ are continuous at $x_i = L_{i1}$ and L_{i2} ,
- (iii) if $\mathcal{G}_{ij,\bar{\omega}}(x_i, x'_j) = A_{ij} e^{\bar{\omega} x_i / v_W} + B_{ij} e^{-\bar{\omega} x_i / v_W}$ for $0 < x_i < \min(x'_j, L_{i1})$, $\delta_{ij} + L_{i1}(1 - \delta_{ij})$ (v_W is the velocity in the wire region), then $B = -M A$, where M is the current splitting matrix at the junction [14–23].

The boundary condition in (iii) arises from the fact that the incoming and outgoing currents (and hence the bosonic fields) at the junction are related by the matrix M . Various constraints at the junction such as current conservation and unitarity of the evolution of the system in real time (i.e., no power is dissipated exactly at the junction) imply that each row and column of M must add up to unity and that M must be orthogonal. The possible M matrices are restricted to two classes parameterized by a single parameter θ [14–23]: (a) $\det(M_1) = 1$ and (b) $\det(M_2) = -1$, which can be expressed as:

$$M_1 = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} b & a & c \\ a & c & b \\ c & b & a \end{pmatrix}, \quad (19)$$

where $a = (1 + 2 \cos \theta)/3$ and $b(c) = (1 - \cos \theta + (-)\sqrt{3} \sin \theta)/3$. We note that $(M_2)^2 = \mathbb{1}$ for any value of θ ; this relation will be used below.

Note that by introducing the orthogonal matrix M , we have made the simplifying assumption that there is no dissipation exactly at the junction. It is computationally simpler to separate the junction, which governs how the incoming currents are distributed amongst the different wires, from the regions of dissipation which lie away from the junction.

Solving eq. (18) with the above boundary conditions and finally taking the limit $\bar{\omega} \rightarrow 0 + \epsilon$, we get the following expression for the dc conductance matrix:

$$G = -\frac{e^2 K_W}{\pi} [\mathbb{1} + M + K_W(\mathbb{1} - M)(\mathbb{1} + \frac{e^2}{\pi} \mathbb{R})]^{-1} \times [\mathbb{1} - M], \quad (20)$$

where \mathbb{R} is a 3×3 diagonal matrix with $\mathbb{R}_{ii} = R_i = r_{i0}(L_{i2} - L_{i1})$; note that R_i is simply the total resistance in wire i . The conductance matrix relates the outgoing current I_i to the potential V_i applied in lead i as $I_i = \sum_j G_{ij} V_j$. One can show in general that each row and column of G must add up to zero; the columns adding up to zero is a consequence of current conservation ($\sum_i I_i$ must be zero), while the rows must add up to zero because each of the I_i must vanish if the V_j 's have the same values in all the wires.

One can show that the conductance of a single wire given in eq. 12 follows from eq. (20) if we choose $M = M_2$ with $\theta = 0, 2\pi/3$ or $4\pi/3$. For instance, if $\theta = 0$, wire 3 decouples from wires 1 and 2 (so that $G_{ij} = 0$ if either i or $j = 3$), while the conductance across wires 1 and 2 becomes independent of K_W and is given by eq. (12), with $R = r_{10}(L_{12} - L_{11}) + r_{20}(L_{22} - L_{21})$ being the total resistance in wires 1 and 2.

Eq. (20) can also be derived in general using the equation of motion approach in the $\omega \rightarrow 0$ limit in the same way as described above for the single wire case. We find that the precise profiles of $K(x_i)$, $v(x_i)$ and $r(x_i)$ in the different wires are not important; all that matters is that the values of K and v are given by K_W , v_W as $x_i \rightarrow 0 + \epsilon$ and by 1, v_F as $x_i \rightarrow \infty$, and that the diagonal elements of \mathbb{R} are given by $R_i = \int dx_i r(x_i)$.

Conductance for the M_1 class.- In the M_1 class, the case $\theta = 0$ is trivial because $M_1(0) = \mathbb{1}$ and $G = 0$. Let us now consider other values of θ . We find that in general G depends on K_W , θ , and the resistances $R_i = r_{i0}(L_{i2} - L_{i1})$. [An exception arises for the case $\theta = \pi$ where we find that G is independent of K_W and depends only on the R_i . This occurs whenever $M^2 = \mathbb{1}$ which is true for $M_1(\pi)$ and also for the M_2 class for any θ as discussed below.] The dependence of G on K_W for the M_1 class is to be contrasted to the case of a single wire where the conductance is independent of K_W [3–7]. In fig. 3 (b), we show the matrix elements G_{11} , G_{12} and G_{13} as functions of K_W for the case of M_1 with $\theta = 2\pi/3$ and $\mathbb{R} = 0$.

In the limit that $R_i \rightarrow \infty$ (which is physically relevant when $R_i \gg \pi/e^2$), we find that the conductance matrix

takes the simple form

$$G = \frac{1}{R_1 R_2 + R_2 R_3 + R_3 R_1} \times \begin{pmatrix} -R_2 - R_3 & R_3 & R_2 \\ R_3 & -R_1 - R_3 & R_1 \\ R_2 & R_1 & -R_1 - R_2 \end{pmatrix}, \quad (21)$$

which is independent of both K_W and θ . Interestingly, the form in eq. (21) is exactly the same as that obtained for a classical system in which three wires with resistances R_i meet at a junction. If a potential V_i is applied to wire i , and the potential at the junction is V_0 , then the outgoing currents are given by $I_i = (V_0 - V_i)/R_i$ for all i . Using Kirchoff's circuit laws to eliminate V_0 , we find that the conductance matrix relating I_i to V_j is given by eq. (21).

Conductance for the M_2 class.- In this case the property $M^2 = \mathbb{1}$ combined with eq. (20) can be used to prove that G is independent of K_W for any choice of θ and R_i . The exact expression for G turns out to be

$$G = -\frac{e^2}{\pi} \frac{3(\mathbb{1} - M_2)}{D}, \quad \text{where } D = 2(\varrho_1 + \varrho_2 + \varrho_3) + \cos \theta (\varrho_1 + \varrho_2 - 2\varrho_3) - \sqrt{3} \sin \theta (\varrho_1 - \varrho_2), \quad (22)$$

where $\varrho_i = 1 + (e^2/\pi)R_i$. We can see that G does not depend on K_W .

Time reversal invariance.- It is interesting to look at our results from the point of view of time reversal (\mathcal{T}) invariance. There are two sources of \mathcal{T} breaking in our system:

- (i) the presence of resistances (i.e., dissipation) clearly violates \mathcal{T} . This is evident from eq. (4) which is not invariant under $t \rightarrow -t$.
- (ii) the current splitting matrix M is \mathcal{T} invariant only if it is symmetric. This is because the outgoing and incoming currents near the junction satisfy $I_{out} = M I_{in}$, while \mathcal{T} interchanges I_{in} and I_{out} . We then see that $I_{in} = M I_{out}$ is satisfied only if $M^{-1} = M^T = M$.

We observe that M_2 is symmetric and therefore \mathcal{T} invariant for all values of θ , while M_1 is \mathcal{T} invariant only if $\theta = 0$ or π . A junction described by M_1 can exist only if \mathcal{T} is broken, for instance, by applying a magnetic field through the junction, assuming that this has a finite cross-section.

The two sources of \mathcal{T} breaking mentioned above are not related to each other since one acts at the resistances and the other acts only at the junction. However, we showed above that if M is symmetric, i.e., is \mathcal{T} invariant, then the conductance G is independent of K_W , regardless of the values of the resistances. Thus there is a remarkable connection between \mathcal{T} breaking at the junction and the dependence of G on the interaction parameter K_W .

Discussion.- To summarize, we have presented a phenomenological formalism which allows us to study the effect of resistive regions in a quantum wire using the language of bosonization. This enables us to calculate the

conductance of systems in which both the interaction parameter K and the resistivity r vary with x . The bosonic equation of motion makes it possible to visualize what happens when a charge density pulse is incident on the resistive and interacting regions. Finally, by introducing a current splitting matrix M to describe a junction, we have extended the analysis to a three-wire system. We find that G depends on K_W for the class M_1 (except for the special case with $\theta = \pi$), but not for the class M_2 in which case we have found an analytical expression for G . Thus we have generalized the well-known results of Safi-Schulz and Maslov-Stone to include systems with junctions and resistances. It may be possible to test our results experimentally by, for instance, varying K_W and θ by applying a gate voltage and a magnetic field near a junction of quantum wires [18, 20], and measuring how this changes the conductance matrix.

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